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A survey of recent grinding wheel topography models

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Abstract

This paper provides a survey of grinding wheel topography models. Recent 1D, 2D, and 3D models are reviewed, and the important model components for a state-of-the-art 3D topography model are identified. Future directions for topography modeling are recommended and, based on this survey, a general modelling approach using grain size, shape, arrangement, and wheel dressing is proposed.

Keywords: Grinding wheel; Topography; Modelling

1. Introduction

The modern grinding process of forming materials has existed for over a millennium, stretching back to water-driven stone wheels; however, as an engineered manufacturing process, it was not until the mid-1940's that grinding was scientifically examined and fledgling mechanics were first applied [1]. Much research in the past half-century has significantly advanced the field, and grinding today is a vital economic constituent in many industrialized countries. Evidence of this can readily be seen by the proportion of grinding machines in metalworking and fabrication plants in the United States. In 1989 *American Machinist* [2] found that in a survey of all the metalworking and metalworking-related machine tools that just less than 25% of all tools were of the grinding-type. Moreover, 13% were dedicated grinding machines (excluding honing and lapping). Merchant's monograph [3] of the machining and grinding research in the past sixty years estimated the annual direct labor and overhead costs of metalworking operations at this time at, conservatively, \$136 billion. When compared to the 1989 US GDP, dedicated grinding was an \$18 billion industry.

Due to its economic importance, significant research effort has been devoted to the optimization and enhancement of the grinding process. Numerical and analytical modelling has proven to be an indispensable tool in this venture. One key aspect of grinding that needs to be addressed when modelling the grinding process is the influence of the wheel surface condition (topography) on the process. There are two main types of topography models: empirical and physical. Empirical models use parameters that are loosely based on physical processes and are computed through statistical regression or optimization of experimental data that generally require little computational expense. Conversely, physical models use parameters that are independent of the application (i.e. material constants, etc.). The majority of models are of the empirical nature which was a result of limited computational resources in the past; however, the new emerging trend in current grinding topology is the development of increasingly physical (or mechanistic) models. Physical models are preferable since more insight is needed into how the wheel surface condition affects the grinding performance [1].

The last time a comprehensive survey of topography models was published was in 1992 [4]. Since then considerable advances have been made in this area. The purpose of this paper is to report on specific recent models developed since 1992 that have shown promising results as well as to point to the future direction of grinding wheel topology modeling. A brief description of topographic models prior to 1992 has been included for historical context and completeness. In addition, the basic components of a state-of-the-art 3D topographic model are identified and a general physical topography modelling approach is presented.

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Nomenclature

$A_{1,2,\ldots}$	Empirical constants
A_{gw}	Cutting edge shape constant for grinding wheel
A_{int}	Grain-diamond dresser intersected area
a	Depth of cut
b	Width of cut
b_{dd}	Dressing diamond characteristic width
c_1	Static cutting edge density per profile depth
c_2	Static cutting edge density
c_3	Static cutting edge density on bond level
C_{gr}	Grain concentration
D	Fractal dimension
d_g	Grain diameter
$\bar{d}_g$	Average grain diameter
d_{eq}	Equivalent grain diameter
f^{dr}	Dressing feed
f_s	Survival fraction of dressed grains
$\mathbf{G}_{ijk}$	Grain position vector
h'	Fracture amplitude
M	Grinding wheel mesh or grit number
m	Empirical exponent
n	Empirical exponent
N_{kin}	Kinematic cutting edge areal density
N_{st}	Static cutting edge areal density
$P(x)$	Probability function
q	Speed ratio
$r_{1,2,\ldots}$	Empirical constants for cutting edge shape
r_g^x, r_g^y, r_g^z	Ellipsoidal axis radii
S_g	Inter-grain spacing
V_g	Grain volume fraction
V_g^{cub}	Volume of grains in cubic cell arrangement
V_{cell}^{cub}	Volume of simple cubic cell
v	Wheel speed
x, y, z	Wheel coordinate directions
x_g^c, y_g^c, z_g^c	Grain center location
z^{dm}	Diamond dresser profile height
z^{gr}	Grain height
z^{fr}	Grain fracture height
α	Dressing effect parameter
$\hat{\alpha}$	Fracture angle random number
γ_0	Variance
$\hat{\delta}$	Grain spacing random number
ζ	Damping factor
H	Noise function
κ	Rake angle
Λ	Total parsed profile length
λ	Scale length
σ_g	Standard deviation of grain sizes
$\Phi(x)$	Normal distribution function
X	Sample data
$\hat{\omega}$	Fracture frequency random number
ω_n	Natural frequency

2. Topography models prior to 1992

2.1. One-dimensional topography models prior to 1992

Very early in the development of grinding modelling research it was well-known that an accurate description of the wheel surface topography was needed. Due to its stochastic nature and experimental difficulties, however, topography modelling was restricted 1D statistical wheel characterizations such as surface roughness of wheels and number of cutting edges. Tönshoff [4] cites Peklenik [5] as the first theoretical study into the cutting actions of grains. This research concluded that the number of cutting edges of the grinding wheel is an inherent characteristic and that any given grain may have multiple edges. Subsequently, Verkerk [6] reported that adjacent cutting edges may be considered a single cutting edge because they lack sufficient chips clearance to act independently. This led to the common-place distinction between 'static' and 'kinematic' grains. The static number of cutting edges is the sum of all of the grains (one edge per), while the kinematic (or active) number of cutting edges is the sum of only the grains that take part in chip formation. This important realization that not all grains participate in material removal set the path of early topography modelling from the mid 1960's until the late 1980's.

Tönshoff [4] produced a comprehensive survey of topography models prevalent in European grinding research up until 1992. At the time, the singular focus of these models was the development of empirical formulae that estimated the static and kinematic number of cutting edges for a given wheel. As such, all of these models were of the 1D type, electing for stochastic rather than physical representations of the topography. Four basic factors used in early 1D topography models were identified. These factors are the cutting edge shape (SF), speed ratio (SR), depth of cut (DC), and grain size (GS). Thus, the basic form of the kinematic cutting edge density is

$$N_{kin} = (\mathrm{SF})(\mathrm{SR})(\mathrm{DC})(\mathrm{GS}). \quad (1)$$

The static grain count density is a function of the volumetric density (grains per unit volume), adjusted by the height of the surface profile. This volumetric density of cutting edges is typically quantified through experimental measurement of the wheel profiles using various methods such as profilometry or optical methods. Tönshoff [4] provided a basic model for static and kinematic grain counts that encompassed many others, namely

$$N_{st} = c_1 z^{A_3} \tag{2}$$

and

$$N_{kin} = A_{gw}\left(\frac{a}{q^2 d_{eq}}\right)^{A_2/2}. \tag{3}$$

The variable c_1 is the volumetric static cutting edge density, a is the depth of cut, d_{eq} is the equivalent grain diameter, and q is the speed ratio. Table 1 shows the configurations for each of the basic factors as well as the static cutting edge formula for several models. Note how each can be simplified into the basic model.

2.2. Two-dimensional topography models prior to 1992

Interestingly, in North America in the 1970's topography modelling departed from the estimations of static and kinematic grain counts. Meyers [12] and Peklenik [13] used autocorrelation theory to characterize the wheel using profile derivatives and profile slopes, respectively, with limited success. Later, researchers had better results employing discrete time series modelling techniques that used very few parameters. Of these models, McAdams [14] used Markov Chain theory, Stralkowski et al. [15] used autoregressive (AR) models, and DeVor and Wu [16] employed autoregressive moving average (ARMA) techniques to develop a discrete estimation of the topography. Pandit and Wu [17] improved upon the early advanced statistical modelling approaches of many other researchers in an attempt to characterize the actual profile of the wheel surface using very few parameters. The model, consisting of a second order differential equation resembling a forced damped vibration system, was fit to a measured profile using regression principals. This system was then transformed using Markov Chain theory into a continuous function [18]

$$\frac{d^2X(t)}{dt^2} + 2\zeta\omega_n\frac{dX(t)}{dt} + \omega_n^2X(t) = H(t), \tag{4}$$

where $X(t)$ is the sampled data, $H(t)$ is a noise function, ζ is the damping factor, and ω_n is the natural frequency. The model allows for any given profile to be characterized by three parameters, damping and natural frequency. Good results were obtained in the study, where each measured profile was successfully fit to the model with these parameters. Although Deutsch and Wu [19] provided a phenomenological assessment of the model parameter-wheel constituent relation, the model lacked direct mechanical ties to the topography.

3. One dimensional topography models after 1992

The 1D construct used for topography models is, by definition, incapable of providing topographical details of the surface. Significant success from a process- and machine-control perspective has been obtained by characterizing the surface by a parameter, be it surface roughness, number of active cutting edges [20], etc. Most recently, fractal theory has been applied to wheel topology [21] utilizing the fractal dimension and has produced some promising results.

Fuzzy logic models, such as Ali and Zhang's [22], have recently been applied to grinding modelling with significant success. These models provide the foundation for intelligent control of grinding processes, and the reader is referred to Rowe et al.'s [23] review for further applications. In this survey, the model selection was restricted to those that take a mechanistic viewpoint of the topography, rather than a machine-control perspective. For this reason, fuzzy logic modelling has been excluded.

3.1. Liao model

Fractal theory-based topography models have garnered much attention in the grinding research community since the inception of fractal theory by Mandelbrot [24]. Models of this type attempt to characterize the wheel profile by a single parameter which has significant implications for machine-control applications. It has been shown that fractal theory can describe engineering surfaces quite well and has been applied to ground surface topographies [25,26]. Most recently, Bigerelle et al. [27] have characterized the paper-ground surfaces using a two-parameter fractal model with good results. These models provide a concrete validation for the application of fractal theory to grinding wheel topology.

Table 1
Cutting edge density factors for 1D models (after [4])

Model reference	Shape factor (SF)	Speed ratio factor (SR)	Depth of cut factor (DC)	Grain size factor (GS)	Static grain count (N_{st})
Kassen (1969) [7]	$(1.51(c_2^2/\kappa x))^{1/3}$	$(1/q)^{1/3}$	$a^{1/6}$	$(1/d_{eq})^{1/6}$	$c_2 z$
Werner (1971) [8]	$\left(2A_1\frac{c_1^{m/n}}{r_1}\right)^{(m/(n+1))}$	$(1/q)^{(m/n+1)}$	$a^{(m/2(n+1))}$	$(1/d_{eq})^{(m/2(n+1))}$	$c_1 z^n$
Lortz (1975) [9]	A_1	$(1/q)^m$	a^n	$(1/d_{eq})^n$	$\int_0^z c_1 dz$
Treimel (1975) [10]	$(A_1(N_{st}/z_{max}))$	$(1/q)^{(1/n+1)}$	$a^{(1/2(n+1))}$	$(1/d_{eq})^{(1/2(n+1))}$	nz
Yegenoglu (1986) [11]	$[2^{(1/r_1+1)}A_1(r_1 + 1/c_3 + r_2)^{(1/r_1+1)}]$	$(1/q)^{(1/r_1+1)}$	$a^{(1/2(r_1+1))}$	$(1/d_{eq})^{(1/2(r_1+1))}$	$c_3 z$
Tönshoff (1992) [4]	A_{gw}	$(1/q)^{A_2}$	$a^{(A_2/2)}$	$(1/d_{eq})^{(A_2/2)}$	$c_1 z^{A_3}$

In essence, fractal theory measures the geometric self-similarity, or the degree to which a particular pattern is scaled and repeated in a structure. With regards to the grinding wheel surface, this approach seems appropriate since the abrasive grains are generally geometrically similar and linear scales of one another.

Liao [21] developed a fractal characterization of diamond grinding wheel topography which is encapsulated by a single parameter, the fractal dimension D. This parameter is determined by parsing measured wheel profiles with various scale lengths λ as shown in Fig. 1. As the scale length increases, more of the profile detail is lost. A logarithmic plot of the total profile length Λ versus the scale length demonstrates fractal behavior if a linear relation with a negative slope is found [21]. The fractal dimension is defined as the slope of this logarithmic plot:

$$D = 1 - \frac{\log(\Lambda)}{\log(\lambda)}. \tag{5}$$

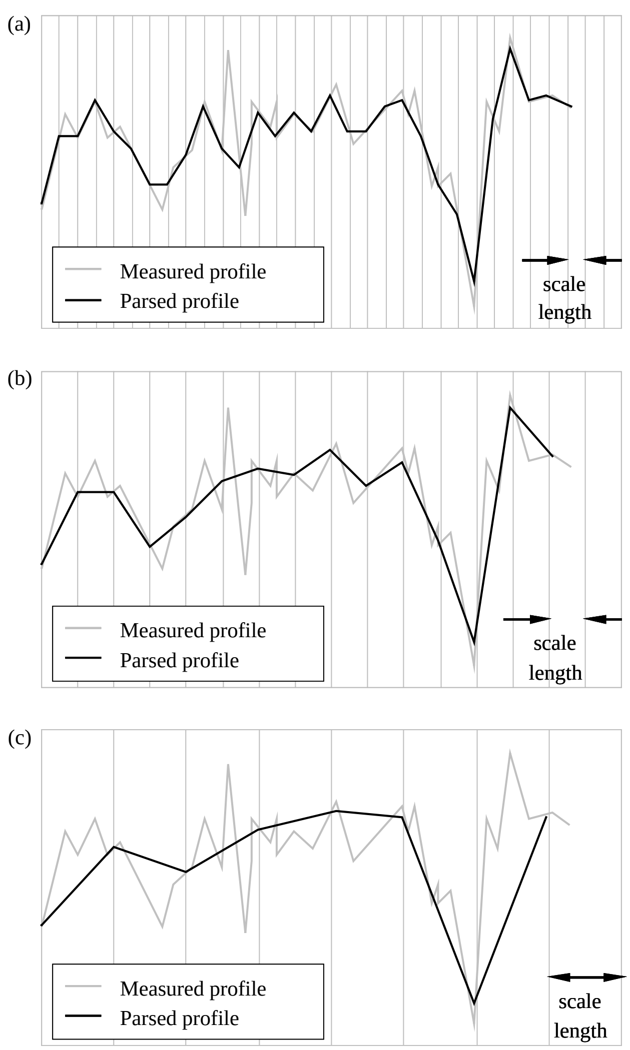

Fig. 1. Schematic of parsed grinding wheel profile with (a) fine, (b) medium, and (c) coarse scale lengths.

It should be borne in mind that fractal theory remains a 1D treatment of grinding wheel topography, albeit with more information. The fractal dimension does not explicitly characterize the profile as a 2D or 3D model would, but acts much like an average surface roughness. It does, however, provide insight to the topographical relationships of the abrasive grains and how similar to one another they are. Fractal theory topography models appear to be an emerging field and may provide interesting results in the future.

3.2. Hou and Komanduri model

Hou and Komanduri [20] developed a model using stochastic approaches to approximate the grinding wheel topography. The model attempts to determine the total number of grains of a certain size within the wheel, rather than a physical mapping of the surface.

The model assumes that grains of different sizes rest on the nominal wheel surface, shown in Fig. 2, where the range of grain sizes is related to the wheel marking. The grain size listed on a wheel's standard marking corresponds to the sieve used to sift the grains prior to fabrication; however, there is a range of possible grain sizes for a given wheel [20]. For example, the opening on a standard #60 sieve is 0.255 mm, and a #54 sieve is 0.291 mm. When grains are first sifted (sorted) for a 54-grit wheel, they are passed through the #54 sieve first, which selects grains of maximum diameter 0.291 mm. Next, these grains are passed through the next finer sieve (#60). This eliminates any grains smaller than 0.255 mm; therefore, the range of grain size for a 54-grit wheel is 0.291–0.255 mm. By applying a normal distribution $\Phi(x)$ over this range, as shown in Fig. 2, the probability of encountering a grain of size d_g may be computed. The normal distribution $\Phi(x)$ is given by

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x^2\right), \tag{6}$$

and the probability of a grain of size d_g, is given by

$$P(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x^2\right) dx. \tag{7}$$

Classically, the distribution is represented as a function of the non-dimensional x parameter. While it is possible to redefine Eqs. (6) and (7) in terms of the grain size d_g it is convenient to simply define a linear transformation rule $x(d_g)$. From Eq. (7) the probability of the total number of grains being exactly of size d_g is known. Using the grain volume fraction V_g the total number of grains per unit area is given as:

$$N_{st} = \frac{V_g^{2/3}}{\bar{d}_g^2}. \tag{8}$$

Using Eq. (8), the number of grains of size d_g passing through the grinding zone per second is

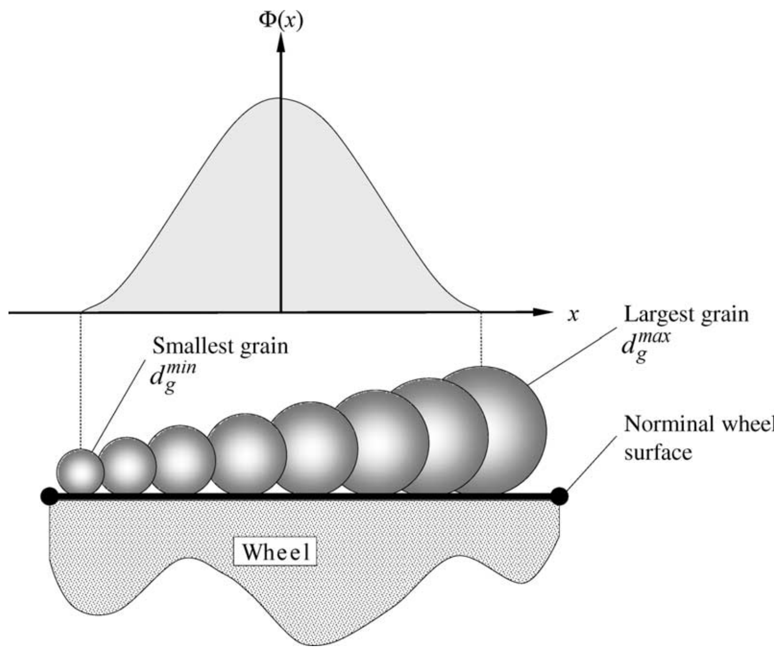

Fig. 2. Schematic of Hou and Komanduri model grain size distribution.

$$\dot{N}^{x}_{\mathrm{st}} = P[x(d_{\mathrm{g}})](vbN_{\mathrm{st}}). \tag{9}$$

The active number of cutting grains N_{kin} is computed by defining a minimum grain diameter that will engage the workpiece. This model does not incorporate dressing effects but provides an excellent example of stochastic modelling of grain sizes and the subsequent static and kinematic grain counts.

4. Two dimensional topography models after 1992

models are a departure from the 1D kind in that the actual shape of the grains is modelled geometrically rather than by an empirical factor. By modelling grains geometrically, the physical effects of dressing, grain location, and others may be studied. The models proposed by Chen and Rowe [28] and Koshy et al. [29] used a physical approach, whilst Torrance and Badger's [30] model employed a stochastic approach.

4.1. Chen and Rowe model

Chen and Rowe [28] developed a topography model that assumed uniform, spherical grains randomly arranged in bond material, and that were subsequently dressed. A representative section of the wheel is modeled by uniformly arranging the grains in bond material, randomizing their location, and subsequently performing a dressing operation. Fig. 3 shows the initial uniform arrangement of grains using a simple cubic (SC) unit cell. Each grain position $\mathbf{G}_{ijk}$ has a grain of size $\bar{d}_{\mathrm{g}}$ which is the average grain size given as [31]

$$\bar{d}_{\mathrm{g}} = 15.2\,M^{-1}, \tag{10}$$

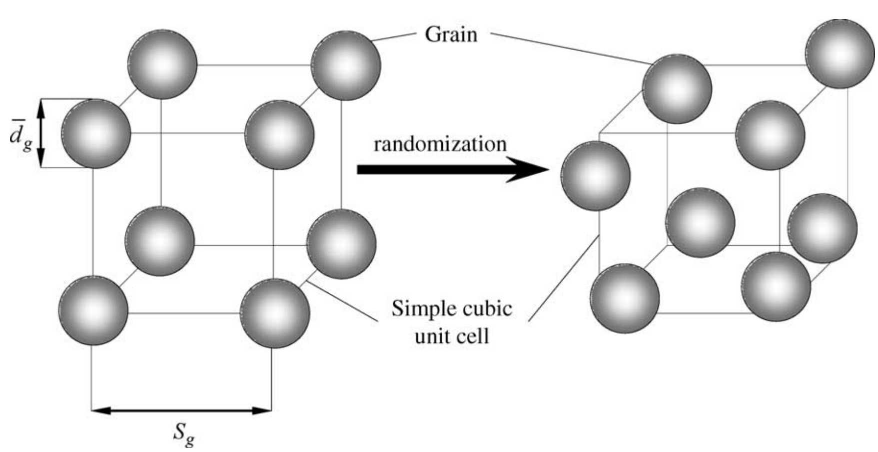

Fig. 3. Grain distribution for a simple cubic unit cell.

where M is the grit or mesh number of the wheel. The total volume of grains in a cubic arrangement V_g^{cub} is

$$V_g^{cub} = \frac{1}{6}\pi\bar{d}_g^3, \tag{11}$$

and the total volume of the unit cell is given by

$$V_{cell}^{cub} = S_g^3, \tag{12}$$

where S_g is the inter-grain spacing. The packing density V_g of the grains is, therefore, the ratio of these two volumes, namely

$$V_g = \frac{V_g^{cub}}{V_{cell}^{cub}} = \frac{\pi\bar{d}_g^3}{6S_g^3}. \tag{13}$$

Note that from Eq. (13), the maximum theoretical packing density for a cubic arrangement occurs when the grains are just touching, or $\bar{d}_g = S_g$, which results in a packing density of 0.524. This packing density represents an upper limit of the model since higher grain densities cannot be arranged using cubic unit cells. It is interesting to note that the maximum packing density according to the ANSI B74.13-1982 wheel marking standard is 0.640, which requires a body-centered cubic (BCC) cell structure [32].

To model the randomness of the actual grain locations in the wheel, each grain position $\mathbf{G}_{ijk}$ is shifted through a randomizing translation

$$\mathbf{G}_{ijk} = \begin{bmatrix} G_{000}^x + iS_g + \hat{\delta}_x \\ G_{000}^y + jS_g + \hat{\delta}_y \\ G_{000}^z + kS_g + \hat{\delta}_z \end{bmatrix}, \tag{14}$$

where the indices $i,j,k = \{1,2,K\}$ represent the position labels of the grain and $\hat{\delta}$ is a random number between 0 and S_g. Fig. 3 shows the rearranged grain structure after the randomization translation was applied. The reader should note that the linear translation of Eq. (14) is subject to the constraint whereby no grain may intersect any other grain.

An important aspect of this model is that it recognizes the importance of dressing effects that are known to play a vital role in the performance and characteristics of grinding [31, 33]. Chen and Rowe [29] considered two main mechanisms in dressing: ductile bond cutting and brittle fracture as shown in Fig. 4. Chen and Rowe [28] assumed that due to the random nature of the grains, material properties, and wheel structure, the fractured surface should be irregular. Thus, the model uses a contour shape defined by the periodic function

$$z^{fr}(x) = z^{dm}(x) + h'[\sin(\hat{\omega}x + \hat{\alpha}) + 1], \tag{15}$$

where z^{dm} is the dressing trace. The variable h' is the fracture amplitude, $\hat{\omega}$ is the random fracture frequency, and $\hat{\alpha}$ is the random fracture angle. The random quantities in Eq. (15) reflect the erratic nature of the fracture and ensure that the contour does not conform to the profile of the diamond.

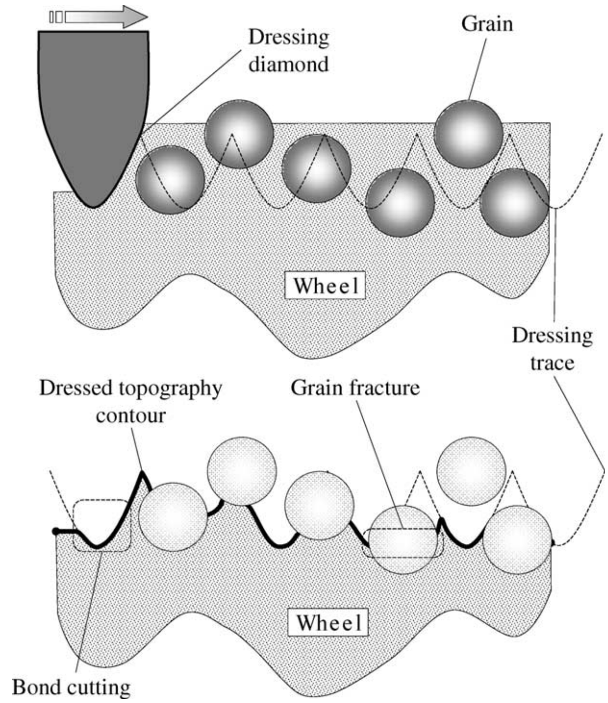

Fig. 4. Schematic of the Chen and Rowe model approximation of the grinding wheel topography.

Chen and Rowe [28] also assumed that the size of the fracture should be proportional to the area of intersection of the dressing tool and the grain as shown in Fig. 5. Therefore, the fracture amplitude was defined as

$$h' = \frac{A_{int}b_{dd}}{4f^{dr}}, \tag{16}$$

where A_{int} is the intersected area of the diamond and grain shown in Fig. 5.

4.2. Koshy et al. model

Koshy et al. [34] developed a 1D topography model for diamond grinding wheels which statistically estimates the average grain protrusion height that was subsequently expanded into a 2D model [29]. The grain size was represented by a normal distribution function, similar to that of Hou and Komanduri's [20] model, and has the form

$$P(d_g) = \frac{A_1}{\sigma_g\sqrt{2\pi}}\exp\left[-\frac{1}{8}\left(\frac{d_g - \bar{d}_g}{\sigma_g}\right)^2\right], \tag{17}$$

where σ_g is the grain size standard deviation given by

$$\sigma_g = \frac{d_g^{max} - d_g^{min}}{6}, \tag{18}$$

and A_1 is an empirical constant. Using Eq. (17), the probability of encountering a grain of size d_g can be calculated as can the average protrusion height. This 1D model was subsequently extended to a multidimensional treatment [30] using a framework similar to that of Chen

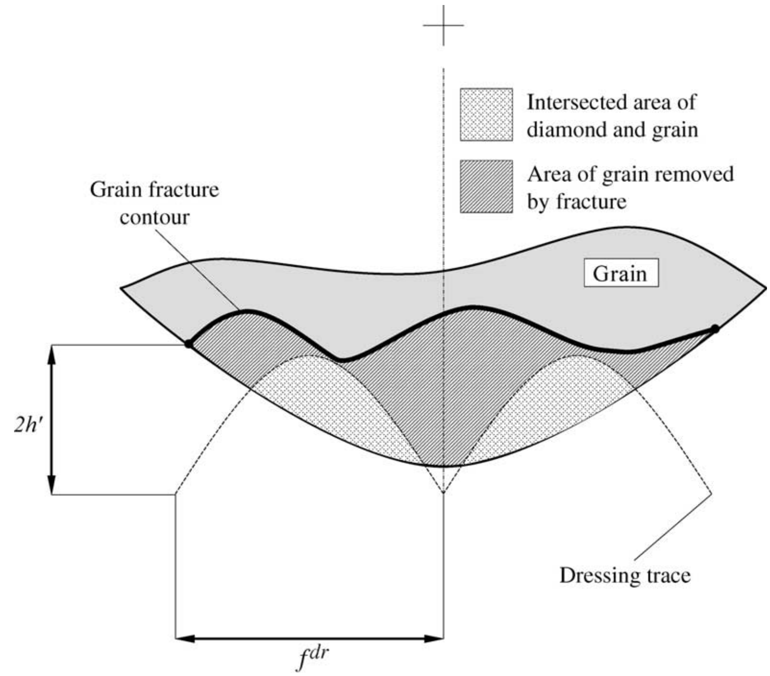

Fig. 5. Grain fracture contour generated by dressing (after [28]).

and Rowe's [28] model with the added advantages of allowing grains to have different diameters. In the multidimensional model the grains' positions are randomized and each grain location $\mathbf{G}_{ijk}$ satisfies the rule

$$\mathrm{dist}(\mathbf{G}_{ijk}, \mathbf{G}_{lmn}) \geq \frac{d_{\mathrm{g}}^{ijk} + d_{\mathrm{g}}^{lmn}}{2}, \quad (19)$$

which ensures that any two grains $\mathbf{G}_{ijk}$ and $\mathbf{G}_{lmn}$ do not intersect one another.

While this model is capable of producing 2D or 3D topographies, it extracts the 1D quantities of protrusion height, static number of cutting edges, inter-grain spacing, and the exposed area of the grains. Although the effect of dressing was not accounted for, it was one of the first multi-dimensional models to incorporate a grain size range treatment.

4.3. Torrance and Badger model

Torrance and Badger [30] developed a grinding wheel topography model that incorporates not only stochastic quantities but also dressing of the wheel surface. The basic premise of the model is that uniform, spherical grains are distributed randomly in the bond material as shown in Fig. 6. Upon dressing the wheel surface, the topography is assumed to take the form of a series of angled surfaces. These surfaces, or slopes, are defined by many parameters but are primarily the product of grain fracture.

The model assumes two predominant forms of fracture occur: bond fracture [35] and grain fracture [31,36,37]. It also accounts for local plastic deformation, or crushing, of the grains. The model begins with the determination of the number of grains per unit area on the wheel:

$$N_{\mathrm{st}} = \frac{6V_{\mathrm{g}}}{\pi \bar{d}_{\mathrm{g}}^{2}}. \quad (20)$$

The model computes the statistically-based fraction or percentage of the grains that survive one dressing pass as

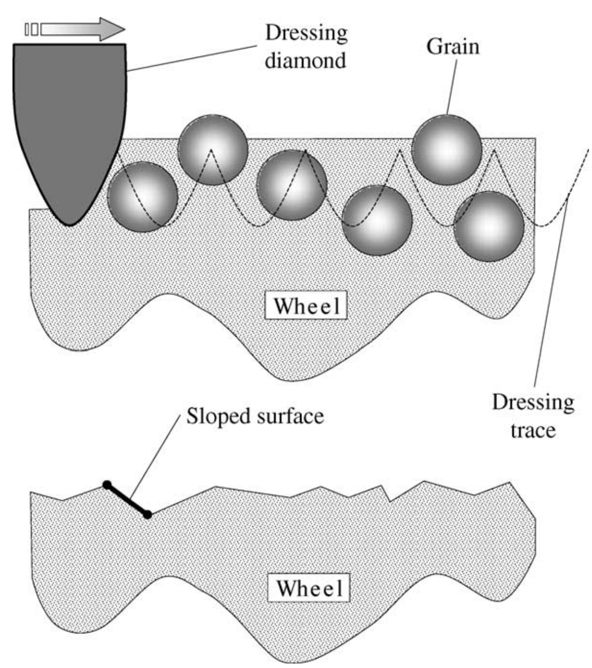

Fig. 6. Schematic of the Torrance and Badger model approximation of the grinding wheel topography.

$$f_s = \frac{1}{\alpha}\left[1 - \frac{1}{\exp(\alpha)}\right], \tag{21}$$

where α is a quantity that incorporates the dressing parameters and grain/bond fracture and deformation. The quantity f_s, in conjunction with the dressing trace, is used to determine what fraction of the grains will be crushed and fractured or dislodged. A crushed grain is assumed to have a linear geometry with a slope of 0.2. Fractured grains are assumed to have a slope of 0.6, and dislodged grains leave a flat void (slope of 0).

Although the formulation represents a 2D profile of a dressed wheel, it does not use this information directly. Instead, a statistical average (rms) slope is used to characterize the surface. Though a physical description of the topography is not used, the model is one of the few multi-dimensional models to assume a topography composed of angled surfaces.

5. Three dimensional topography models

In the previous section the reader may have noted that the initial structuring of the model (grain positioning, shape) was, in fact, three-dimensional. What separates these models from true 3D formulations is the end product. A three-dimensional model is one where not only are the grains considered as three-dimensional objects, but a 3D surface is produced to estimate a measured topography. At the time of this report, the model proposed by Hegeman [38] was the only three-dimensional model that could be found in the literature. This model approximates the grinding wheel topography by a randomized arrangement of three dimensional ellipsoidal grains as did Büttner [39].

Hegeman [38] noted that most grains tend to be ellipsoidal in shape not spherical in shape as evidenced by SEM photographs. In this model the ellipsoids size and orientation can change. Grain rotation is accomplished by varying the axis radii r_g using stochastic distributions. The grain shape function, in the wheel (global) coordinate system, is

$$z^{gr}(x,y) = r_g^z\sqrt{1-\left(\frac{x-x_g^c}{r_g^x}\right)^2-\left(\frac{y-y_g^c}{r_g^y}\right)^2}, \tag{22}$$

where the grain center is located at $(x_g^c, y_g^c, z_g^c = 0)$. Eq. (22) represents the smooth ellipsoid surface protruding from the wheel surface $z = 0$.

In an effort to simulate the non-smooth surface of the grain after dressing the model employs a randomizing function that is a three dimensional generalization of the one proposed by Chen and Rowe [28]. Hegeman [38] defines a stochastic periodic function of the form

$$z^{fr}(x,y) = \cos(\hat{\omega}_x x + \hat{\alpha}_x) + \cos(\hat{\omega}_y y + \hat{\alpha}_y) \tag{23}$$

to simulate the effects of dressing, where $\hat{\omega}_x$, $\hat{\alpha}_x$, $\hat{\omega}_y$ and $\hat{\alpha}_y$ are random numbers. This function adds small deviations to the ellipsoid surface creating a 'rough' texture. Combining Eqs. (22) and (23) yields the total grain shape after dressing:

$$z(x,y) = \begin{cases} z^{fr}(x,y) & \forall\, x,y \text{ outside grain} \\ z^{gr}(x,y) + z^{fr}(x,y) & \forall\, x,y \text{ inside grain} \end{cases}. \tag{24}$$

With Eq. (24), the dressed grain shape and surrounding material can be modeled. For practical application, several grains are assembled to form a surface topography by using randomly sized unit cells as shown in Fig. 7.

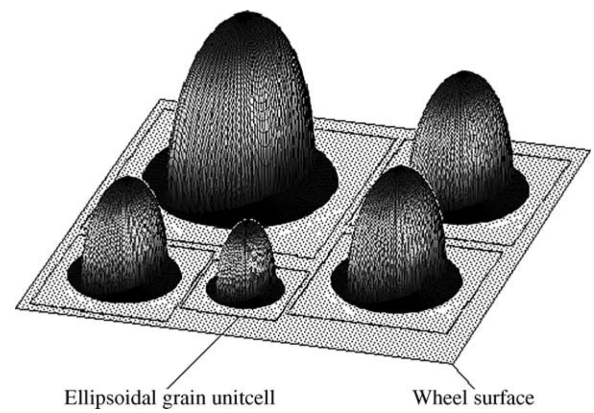

Fig. 7. Schematic of Hegeman model approximation of the grinding wheel topography.

Several wheel parameters (listed in Table 2) were experimentally determined in order to define the unit cells and grain axis radii. Hegeman used confocal microscopy to characterize the grain shape [38]. This method varies the focal length of an objective lens and measures changes in reflected light intensity from a scanning optical microscope and is able to resolve steep slopes and enhanced lateral field width by approximately 20% [40], allowing for estimation of the areal grain concentration C_{gr}.

6. A framework for a general 3D model

The physically-based grinding wheel topography models reviewed in this work contain similar constructs and can be summarized in a general modelling approach. Fig. 8 shows a framework for a physically-based topography model. The framework has two main components consisting of defining

Table 2
Required wheel parameters for Hegeman model

Wheel parameter	Symbol	Experimental technique
Grain concentration	C_{gr}	Scanning electron microscopy
Grain base radius	$r_g^x = r_g^y$	Confocal scanning optical microscopy
Grain protrusion height	r_g^z	Confocal scanning optical microscopy

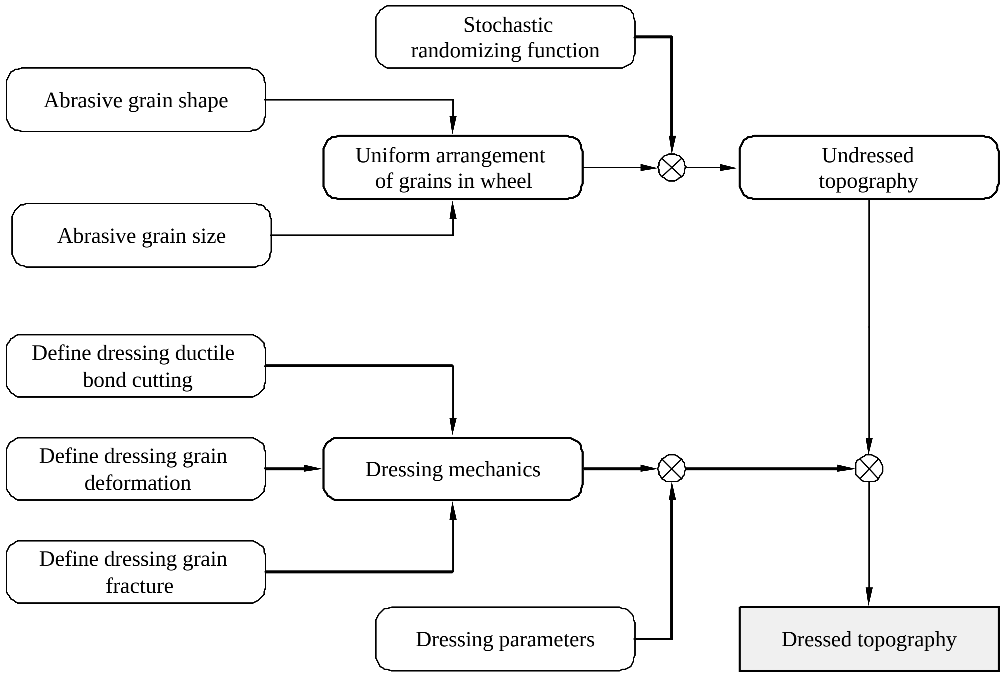

Fig. 8. General 3D physical topography modelling approach.

the undressed topology followed by the application of suitable dressing mechanics.

The undressed topology consists of the grain shape, size, and position. Typically, grains shape and size is modeled as spheres since it is the most convenient shape to use and will be subsequently dressed to produce the cutting edges. However, more complex shapes such as ellipsoids could be used. Next a uniform arrangement of the grains within the wheel is accomplished by assigning a grain location $\mathbf{G}_{ijk}$ to each site of a unit cell. Wheels with a nominal packing density less than 50% can use the simple cubic cell structure (see Fig. 3), while more dense wheels will require a body-centered cubic or similar unit cell. The final undressed topography is produced using a randomizing function or algorithm to translate each grain location $\mathbf{G}_{ijk}$ subject to the constraint that grains cannot occupy the same space to mimic the actual arrangement.

The dressing mechanics are a product of three major mechanisms: grain fracture, ductile bond cutting, and grain deformation. The ductile bond cutting is simply the dressing point removing the bond material and leaving a trace of the tool. The grain deformation and fracture are the primary effects of dressing although grain fracture appears to have the most prominent effect. Grain fracture should relate to the severity of the dressing operation as well as the type of abrasive material used.

7. Conclusions

Several recent grinding wheel topography models have been discussed. The current 1D models attempt to characterize the surface condition by a single parameter. This type of model, while effective for process- and machine-control applications, does not provide much detailed information on of the surface condition. A recent advance has been the use of fractal theory which has shown promising results.

The 2D and 3D models reviewed represent a distinct demarcation point in grinding wheel topology: modelling with a physical perspective. These models consider the effect of wheel structural components and dressing mechanics on the final wheel topography. The application of fundamental mechanical theory (such as grain fracture) renders these models more useful and relevant to the researcher.

A general model was presented (Fig. 8), which was the accumulation of various components from the 2D and 3D models surveyed. The major components identified were:

1. Abrasive grain shape and size
2. 3D, randomized grain arrangement
3. Dressing mechanics accounting for grain fracture and deformation
4. 3D dressed wheel surface.

It appears from the literature that the grain arrangement approach proposed by Chen and Rowe [28] has had much success and relates well to the actual structure of the wheel. The same may be said for the stochastic treatments of the grain size. Though the distributions vary somewhat in the models, the basic principals remain consistent; however, the dressing mechanics, specifically grain fracture, have not been addressed in a detailed fashion.

Future modelling may be well-served to focus on the introduction of brittle fracture mechanics, possibly in 3D, in the dressing component. Appropriate modelling of the ceramic fracture surface of the grain should incorporate high-strain rate effects and damage accumulation. Significant modelling advances in ceramic fracture have been realized by Johnson and Holmquist [41] which may aid in future grain fracture modelling.

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